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ACOUSTICAL AND VIBRATIONAL STUDIES RELATING TO AN OCCURRENCE OF SONIC BOOM INDUCED DAMAGE TO A WINDOW GLASS IN A STORE FRONT

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ACOUSTICAL AND VIBRATIONAL STUDIES

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SUMMARY

An analysis is presented of some acoustical and vibrational phenomena which may be involved in certain types of structures acted on by sonic boom shock waves. Mathematical models are formulated and applied to a specific incident of damage to a window glass in a store front which occurred during the series of sonic boom test flights at Oklahoma City in 1964. Some of the potential conditions for producing magnifications of response when certain tuning relationships exist between structural elements and/or between structural elements and the sonic boom shock wave elements are demonstrated.

General conclusion of the study is that the assumed sonic boom shock wave would have been incapable of producing a critical stress level in this particular window installation. However, several significant facts were unknown and were not determinable such as the actual pressure-time history of the shock wave acting on the structure and window, and the condition of the window installation prior to breakage.

INTRODUCTION

During the series of 1,253 sonic boom test flights conducted in the Oklahoma City area during 1964, an 8' x 10' x 1/4" plate glass window in the store front of a single-story commercial building was broken coincidentally with the occurrence of one of the sonic booms. See Figure 1. The glass breakage was witnessed by several persons and the newspaper accounts of the incident, including statements of these witnesses, are shown in Figure 2.

This particular sonic boom (#690 of the series) occurred at about 1:20 p.m. on Sunday, May 17, 1964 and was produced by an F-101 aircraft at 40,000 feet altitude on a scheduled steady-state course at a scheduled speed of Mach 1.4. The nearest location at which a pressure-time measurement of the boom was recorded was at a location designated as Test House #1. This location was about 5 miles northeast of the store building as shown in Figure 3. A copy of the pressure-time measurement recorded at Test House #1 for this flight is shown in Figure 4.

Using this specific incident of glass breakage as an example, an analysis has been made of vibrational and acoustical factors which may have been involved to determine if any of these factors could have contributed to the failure of this particular window from this particular sonic boom. In a concurrent and related study (Ref. 1) a mathematical method was developed to calculate the pressure-time history acting on the window glass to determine if a shock wave with a pressure-time history such as that recorded at Test House #1 could have been altered significantly by building orientation and configuration. This related study indicated that, theoretically, no abnormal or unusual pressure-time condition would have been produced.

Hence, the approach taken for this study is as follows:

1. Determine all significant physical characteristics and dimensions of the building.

2. Formulate mathematical models of the building taking into account as many factors as possible that could influence the dynamic response of the window. These factors include the acoustical coupling between the ceiling and windows as well as the acoustical coupling with other rooms within the building.

3. Determine the stress to which the mathematical models were subjected in response to an assumed sonic boom. The input to the models was taken to be identical to the signature measured at Test House #1 for lack of any better information.

4. Study the possibility of failure, taking into account the statistical strength of glass, in response to the assumed sonic boom.

Formulation of the mathematical models is based upon measured and calculated data. The natural frequencies of the ceiling and windows were measured with appropriate instruments and the existence of coupling between the ceiling and windows was verified by steady-state vibration tests. The masses of the various elements were calculated after physical dimensions had been tabulated. The elastic properties were calculated from the known natural frequencies and calculated masses.

The response of the lumped parameter system was obtained by two means: 1) the analogue computer, and 2) the digital computer. The analogue computer was used to study specific cases whereas the digital computer was used to solve for maximax values as a function of several parameters. The work done on the analogue computer is presented in the time domain; the results of the digital computer study are presented in the frequency domain.

Since the natural period of the window was high compared to the duration of the assumed sonic boom shock wave, a multi-mode analysis of the window, as mounted in an infinite baffle, was accomplished. It would be impractical to treat the entire building as a continuous system since many assumptions would have to be made regarding the boundary conditions presented by racks of shoes and of flexible partition walls.

Purpose of this paper is intended to advance the development of effective methods for predicting, investigating and evaluating possible sonic boom damage to window glass.

Figure 1 - Store front of the commercial building in which an 8' x 10' x 1/4" window glass was broken coincidentally with occurrence of a sonic boom. Third large pane from right is the location of broken window.

Monday, May 18, 1964
THE DAILY OKLAHOMAN

Boom Hits; Glass Flies At City Store

"The sonic boom went off and then the glass just bulged out," is how a witness described the shattering of a large display window Sunday afternoon at Kinney's Shoe Store, 3718 NW 23.

The 8 by 11-foot plate glass window popped out shortly after 1 p.m. The FAA said a sonic boom did occur at approximately the same time but they refused further comment until they have completed an investigation.

Mr. and Mrs. A. D. Hamilton, 3741 NW 31, were parked in a service station at NW 23 and Portland.

Hamilton said, "I was standing next to the pickup truck looking at the store and wondering if it was open. The boom went off and the glass went flying."

"The glass just bulged out at the same time the boom went off," he said.

A 15-year-old girl, Francine Ann Irvine, told police officer Mike Williams she was just 15 feet from the window when it shattered. The girl said she was window shopping and there was a boom and then a crash.

Mrs. LeRoy Clark, who lives across the street from the shoe store, reported the incident to police.

"I jumped from the sound of the boom and then I heard the shattering of glass," she said.

Tuesday, May 19, 1964
THE DAILY OKLAHOMAN

FAA Checks Claims Boom Broke Window

Federal Aviation Agency engineers are looking into the claim that an 8 by 11 foot plate glass window was broken out Sunday by a sonic boom.

Until the engineer's report is complete and the manager of the store files his claim forms, no decision will be made on the damage claim.

Mark Weaver, public information officer for the FAA, said a structural engineer was sent to Kinney's Shoe Store, 3718 NW 23, Monday morning, where a 375-pound sheet of glass had been broken out the day before.

It 'Bulged Out'

Witnesses said the sheet "just bulged out" at the same time the boom hit, shortly after 1 p.m. Sunday.

Weaver said residents' reports of hearing "double booms" since the tests were resumed last week were probably true.

"Actually there is always a double boom," said Weaver. "One shock wave comes from the wing and one from the tail, but with the F-104 the two were heard simultaneously.

Boom Separated

"With the F-101, which is being used now instead of the F-104, the different shape of the wings and tail gives a separated boom."

Weaver said the booms probably did sound louder, due to the different plane, but that the overpressure remains at about 1½ pounds.

"We intend to throw in a few booms of about two pounds, and gradually build up to where all booms will be at that level," he said.

"Two pounds will be the absolute maximum," he added.

Figure 2 - Copies of newspaper accounts of window damage to store front.

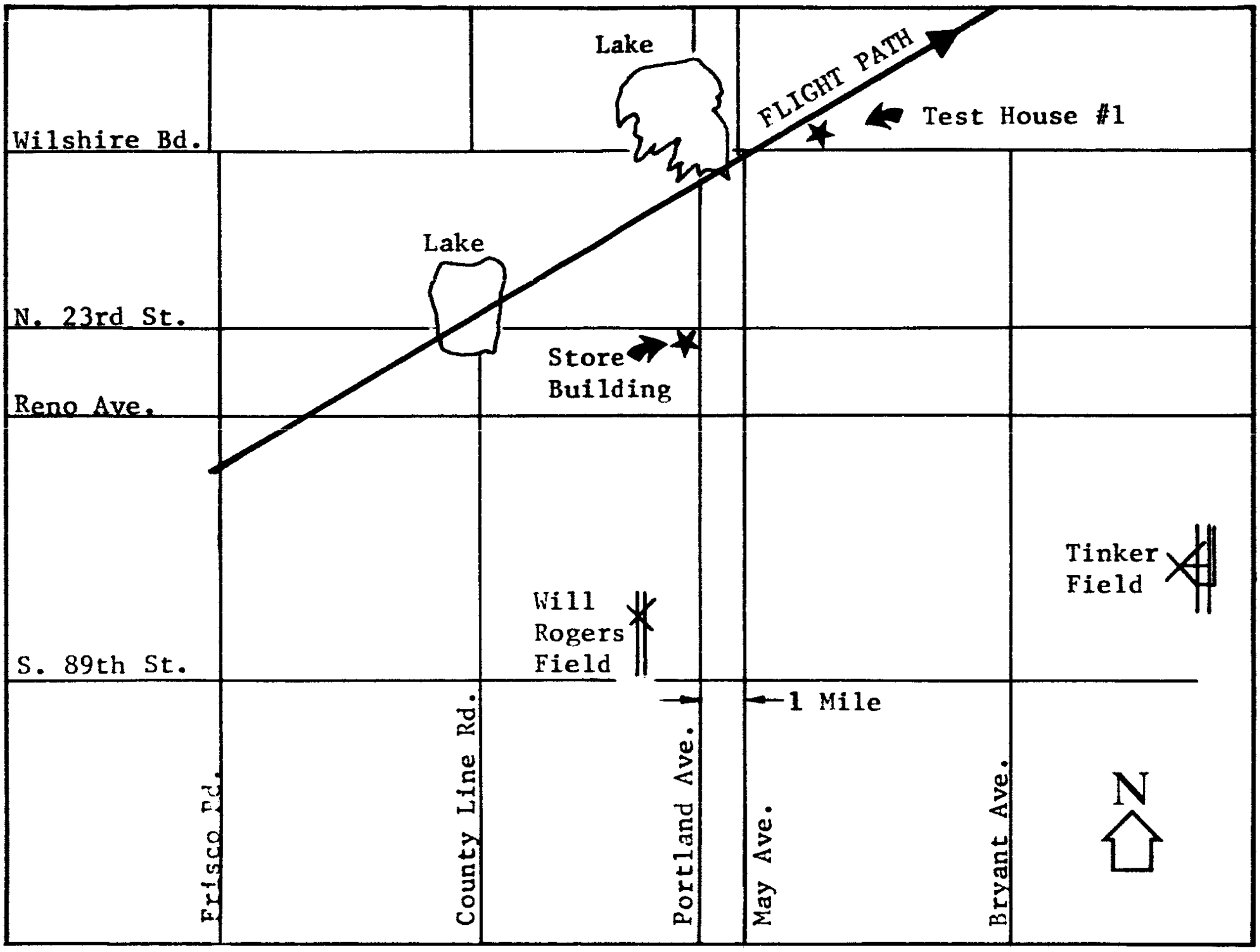


Figure 3 - Oklahoma City area map showing relative locations of store building, Test House #1 and flight path.

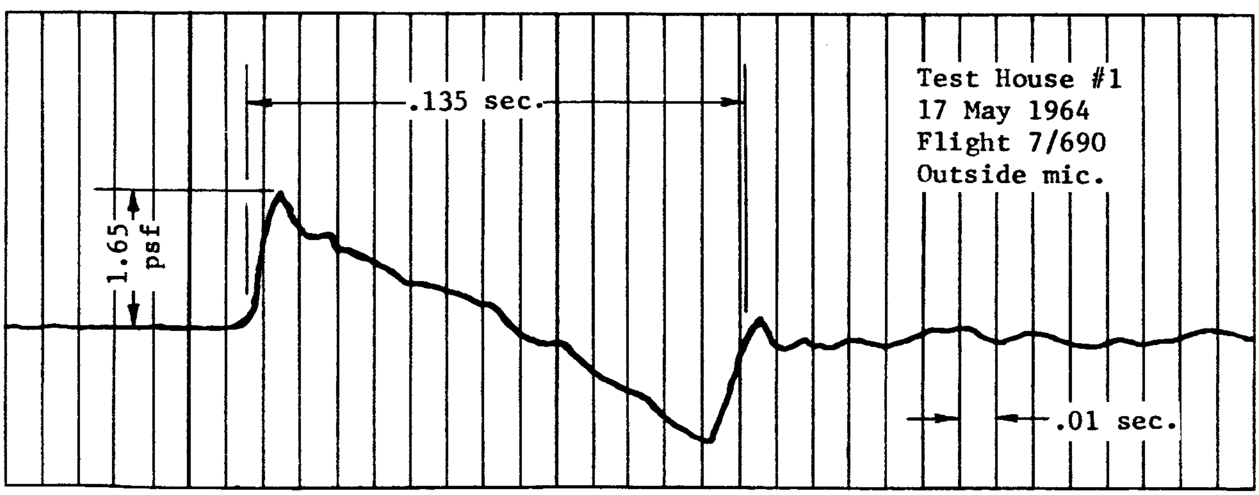


Figure 4 - Tracing of oscillograph tape of pressure-time record measured at a location on the ground in back yard of Test House #1.

SYMBOLS

a	plate length in x direction
A	surface area
b	plate width in y direction
c	speed of sound in air
C	damping coefficient
D	flexural rigidity of plates
E	Young's modulus of elasticity
f	frequency
F	force
F_0	peak force
h	plate thickness
I	second moment of area of cross-section about neutral axis
K	spring constant
K_c	acoustic coupling spring constant
L_e	equivalent neck length of Helmholtz Resonator
M	mass
m	mode number in x direction
n	mode number in y direction
P	pressure
p	natural circular frequency
t	time
T	natural period
V	volume
v	velocity
x	displacement

x	plate length in x direction
y	plate length in y direction
δ	ratio of specific heat for air
λ	Poisson's ratio
ζ	damping ratio
ρ	density of air
σ	tensile stress
σ	standard deviation
τ	duration of force
ω	circular frequency

Subscripts

1,2	denotes mass number
F	forced era
R	residual era
st	static deflection

STRUCTURAL CONSIDERATIONS

Exterior configuration of the store building is shown in Figure 5. The interior is divided into a main room and a U shaped stock room as shown in Figure 6. The two separate volumes are connected by six open passage-ways. The separating wall between the stock room and main room is a curtain wall which is plywood on wood studs. It is doubtful the partition is even moderately airtight.

The building proper is essentially airtight with the exception of an air-conditioning inlet duct of approximately 3 square feet at the rear of the building and two small ventilating openings in the louvers in the gables of the roof. The volume inclosed between the roof and the ceiling joists communicates with the stock room through a 1 inch gap between the masonry walls of the building and the sheetrock ceiling. Hence, the ceiling is not connected to the walls at the east and west ends of the building, but is partially supported by the curtain wall separating the stock room from the main room.

The back portion of the stock room is covered by a relatively flat ceiling which is very flexible and which supports the heavy air conditioning system. The condensing unit to the air conditioner sits upon this flat roof.

Since there is very little communication of the volume inside the building to the atmosphere, it must be concluded that the building probably has no true Helmholtz modes. However, there is a possibility for an internal mode if the air in the passageways between the stock room and main room is considered as the neck and the volume of the stock room is considered as one volume, and the air enclosed by the main room considered as another volume. However, this is considered unlikely because of the existence of many minute air leaks between the two rooms. Also, the curtain wall separating the two rooms serves as storage shelves and appears to be quite flexible. For this reason, the internal Helmholtz mode has not been given serious consideration, although an approximate analysis was made by an analogue computer.

The ceiling-roof assembly consists of a number of identical wood trusses with sheet rock nailed to battens which, in turn, are nailed to the wood ceiling joists as shown in Figure 7. The trusses seem to be very stiff, although they are probably somewhat longer than would normally be used in general architectural practice. The span of each truss is approximately 50 feet, as shown in Figure 7. Measurements made with respect to the floor of the main area indicate that there is at least 1 inch of sag at the center of the room. There are several relatively severe dry wall cracks running east-west in the ceiling of the main room at the approximate center of the 50 foot span. These cracks have been repaired on several occasions but seem to be re-opening again. There is evidence that additional bracing was put into the truss work over the center of the main area in an effort to alleviate this problem. Periodic testing of the wood moisture content of the ceiling-roof structure was maintained during the field testing phase of this study.

The windows in the front of the building are mounted in flexible mullions as shown in Figure 8. Window #6, the left-hand duplicate of Window #3, was disassembled to determine method and general quality of glazing practice for this store front installation. Glazing practice was found to be in accordance with accepted standards. When one window is excited by transient force, the other windows adjacent to it also vibrate. Although individual natural frequencies of the separate panes can be excited in any free vibration recording, the existence of beats can be seen between the adjacent windows. Hence, it is virtually impossible to arrive at an exact mathematical model.

It seems that the best mechanical approximation of the structural system would be to consider the roof and ceiling assembly as one individual mass and the entire assembly of glass at the front of the store being a second mass. The two masses are coupled together by the air in the entire volume of the store and the masses are grounded by their physical elasticities. Thus, if it is assumed that the roof of the structure is excited only in its fundamental mode as is each of the window panes in the front of the store, then an approximate mathematical model would be a lumped 2 degree of freedom system. For this analysis to be correct, it would be necessary for all of the windows to move together in phase acting more or less as one long narrow window. In this case, the properties of the mullions are not known and the edge conditions of the individual panes can only be estimated.

Since to make any type of precise analysis of the structural dynamics of the building would be hazardous, considering the large number of assumptions that must be made, the following lumped parameter analysis is valid only inasmuch as the assumptions are reasonably correct. This is to say that if the store building can be approximated by a 2 or 3 degree of freedom lumped parameter system, then the following mathematical models would predict reasonably well how it might perform. The main emphasis of the mathematical analyses has been in finding the pressure and displacement magnification factors. Obviously, if the lumped parameters system has a large displacement magnification factor, it similarly would have a large stress magnification factor.

The stresses in a rectangular, simply supported plate are readily calculated if the edge conditions are known. However, in this case, the exact edge conditions of each individual pane of glass are not known so it is probably pointless to discuss absolute values of stress except for the case of the plate mounted in an infinite baffle.

There is one other consideration that enters into this analysis at this point. The shock wave actually behaves something like a rolling load across the roof of the building and across the front of the store where the windows are mounted. In all of the following lumped parameter analyses it has been assumed that the wave is at normal incidence and the force is applied suddenly over the entire area. This, of course, will result in a larger value of magnification factor than if the rolling load were considered. However, as mentioned earlier, there are so many other assumptions to be carried on that this is probably of minor consequence in the overall analysis. The analysis used herein should serve to identify the greatest upper bound of system response.

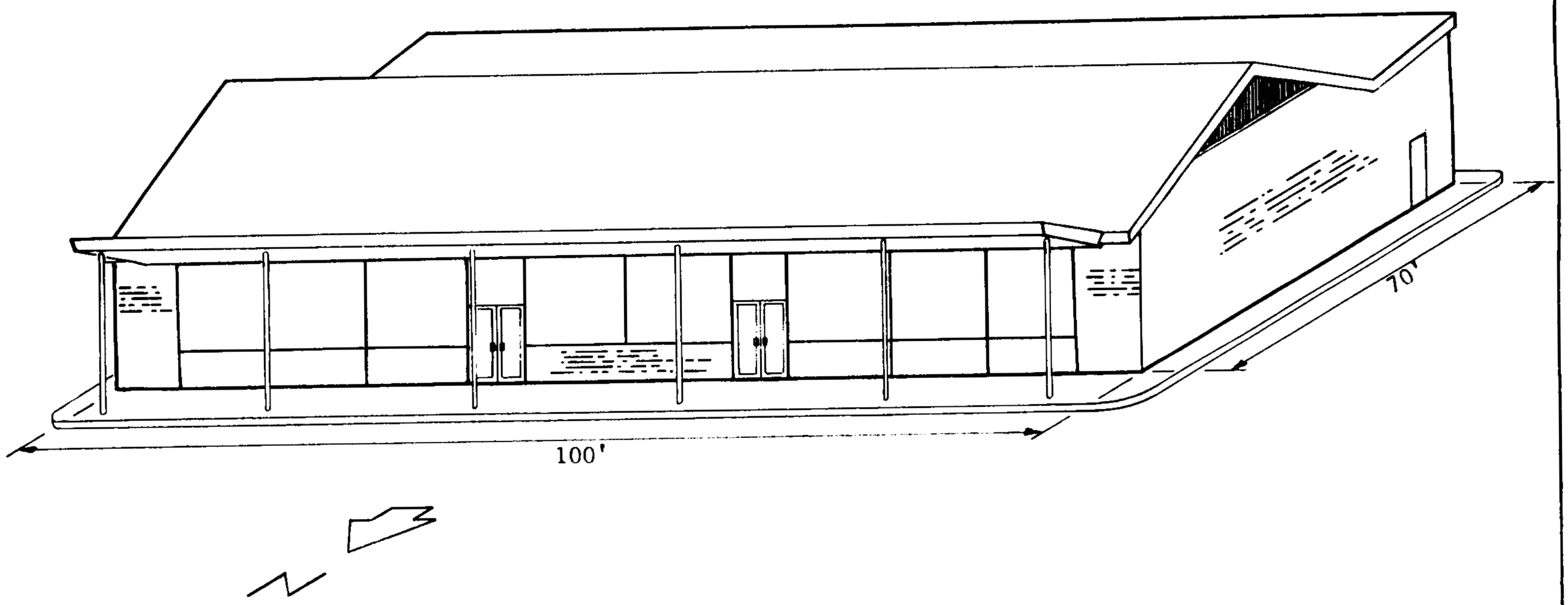


Figure 5 - Exterior configuration of store building.

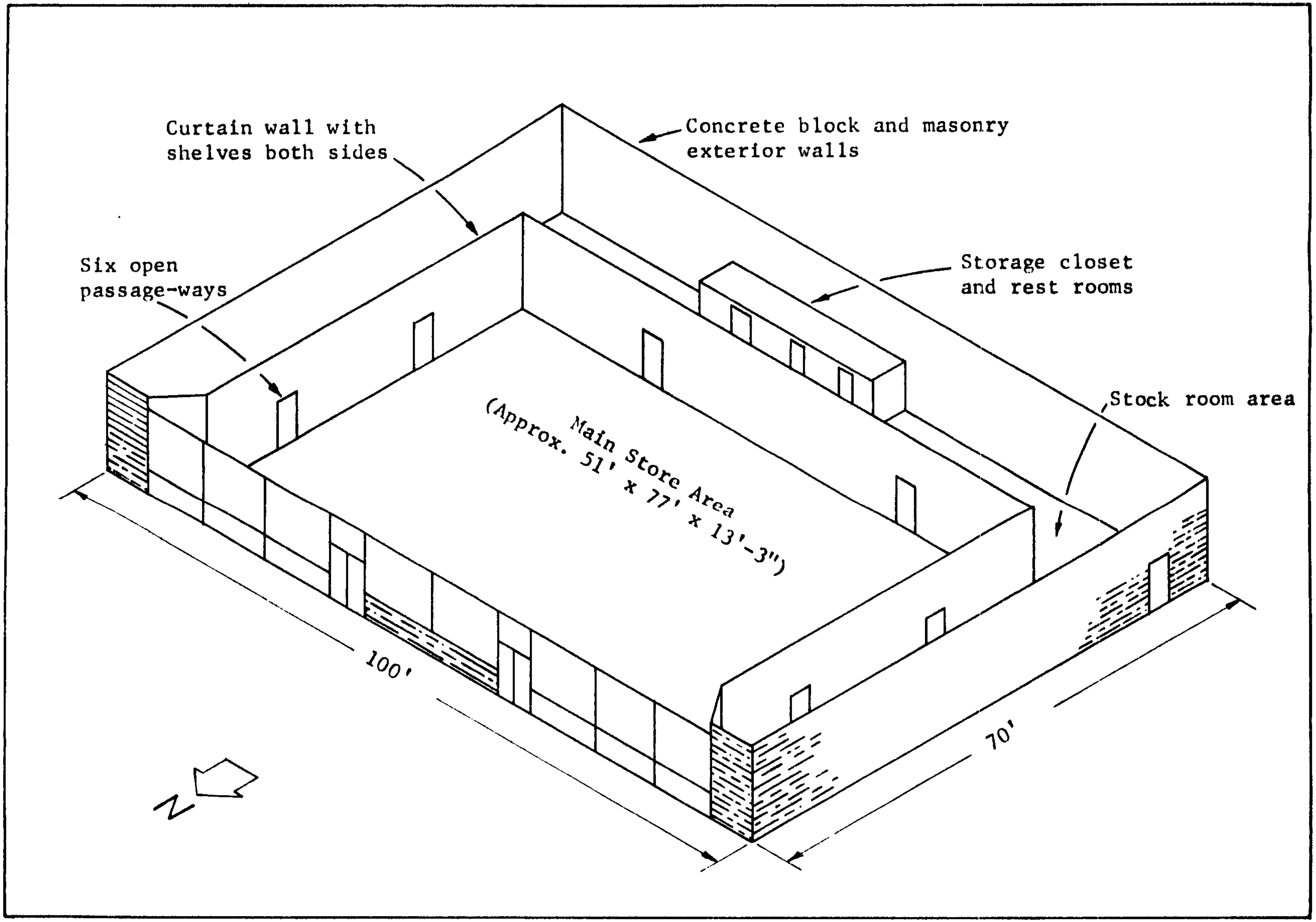


Figure 6 - Interior configuration of store building.

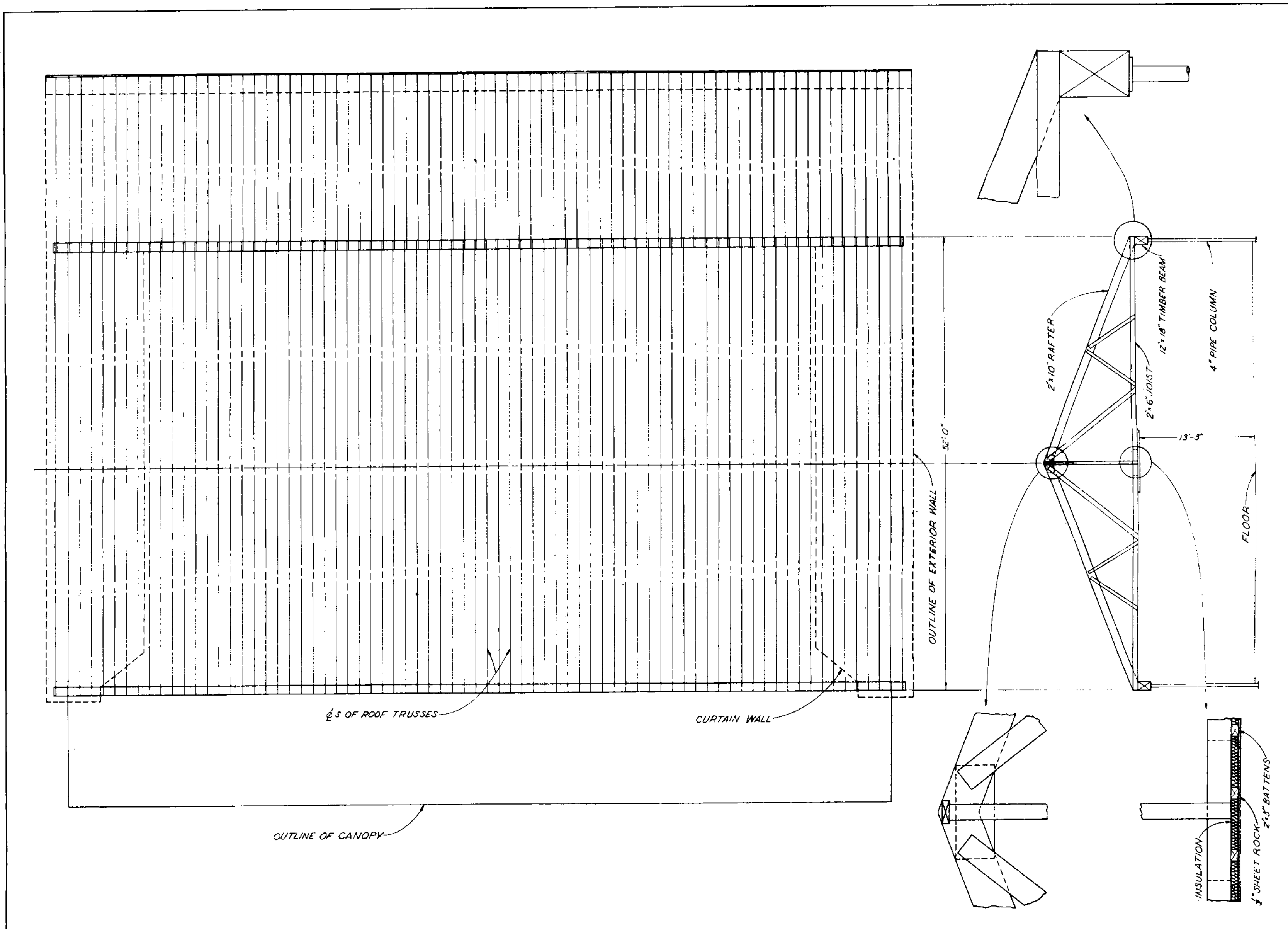


Figure 7 - Details of wood truss structure over main store area.

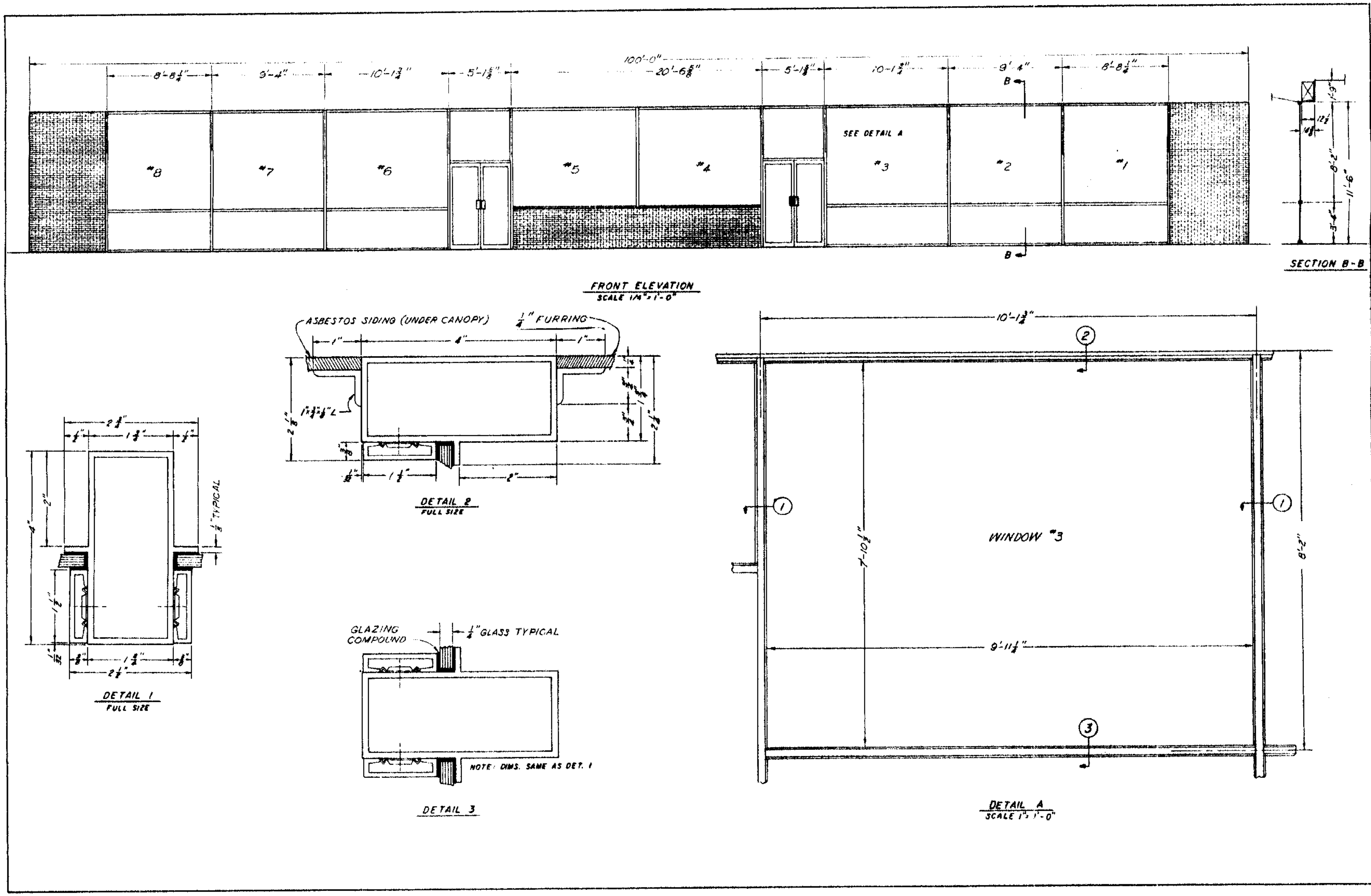


Figure 8 - Store front and window mounting details. Note window reference numbers. Window #3 is location of pane broken on May 17, 1964.

FORMULATION OF MATHEMATICAL MODELS

Natural Frequency Measurements

The natural frequencies of various windows and of the ceiling-roof structure of the building were measured on several occasions. The natural frequency of the ceiling-roof was measured by two methods. First, by attaching a DC differential transformer to the ceiling with the concrete floor as a reference point. Second, by attaching a velocity transducer to the ceiling and feeding its output through a conventional integrating vibration meter. Outputs of both instruments were recorded on a recording galvanometer at high paper speed.

Natural frequency of the windows was measured by differential transformers with care being taken to insure that the core of the transformer was not binding. A rough measure of the damping factor of the windows can also be taken from these recordings although caution must be exercised since the core of a differential transformer can have appreciable friction under certain conditions. To obtain more accurate measurements of the damping factor of windows, a displacement transducer based upon capacitive reactance or inductive reactance would probably be superior. The windows were excited by gently striking them by hand or by gently pushing the glass outward and sharply releasing it. Both methods proved to be effective although if the glass were struck too sharply, higher modes could be excited.

The ceiling-roof structure was excited by a man jumping on the roof at a prearranged signal and recording the ensuing vibration after contact. The instrumentation was sufficiently sensitive that a man's footsteps could be monitored as he walked across the roof. The weight of a man on the roof is small compared to the total weight of the roof structure and it is judged that this additional mass would have little effect upon true natural frequency.

Natural frequencies of the windows, as measured on several different occasions, and as would be expected, did not change with time. There is some question as to the boundary conditions of the windows since all are mounted in aluminum mullions which are quite flexible. It was noted in all of the tests that when any one of the first three windows from the west side of the building was excited by pushing or bumping, the rest of the windows were excited. This suggests that boundary conditions of the glass windows are subject to question since there is definite flexural coupling between adjacent windows. In many of the records, beats were observed which would be the result of exciting two natural frequencies in a coupled system simultaneously.

Measured natural frequencies of the first two windows (#1 and #2) from the west side of the building were both around 5 cycles per second and did not change with time. Window #3 (the window replacing the one that failed) has a natural frequency of 4.10 cycles per second. These values are surprisingly close to the calculated values for the natural frequency of a simply supported plate if the modulus of elasticity is taken to be 10 million pounds per square inch and Poisson's ratio to be .21 with a density of .088 pounds per cubic inch. Calculated value for Window #3 is 4.28 cps.

Although the natural frequency of the ceiling-roof seemed to vary with time, or with wood moisture content, it also was somewhat dependent upon where the jumper on the roof was exciting the roof. In a structure as complex as this, it is logical to assume that there may be several natural frequencies quite close together. Jumping on the extreme end of the roof would produce a higher frequency than jumping in the exact center of the roof. Based upon records made by jumping in the exact center of the roof, or near the north-south centerline of the building, the natural frequency ranged from approximately 9 cps in February down to approximately 8 cps in May. The only atmospheric changes during this period were the temperature and the relative humidity. In February, the wood moisture content was much lower (6.4%) than that in May (9.7%) and it is thought that the decrease in natural frequency is due to a slight increase of the overall mass of the wood roof structure although certain temperature effects could enter in also. The building was the same for the tests in all other respects in that all the doors were closed and no structural revisions had been made. Therefore, the shift in natural frequency, although slight, must be attributed to the change in atmospheric conditions.

Acoustic Coupling Measurements

Two series of tests were run to determine whether acoustic coupling might exist between the windows of the building and the ceiling-roof structure. During these tests the ceiling was instrumented with a velocity transducer fed into an integrator. The window vibration was measured with a differential transformer and the pressure inside the main room of the store was monitored with a special piezoelectric microphone system having a sensitivity of 11.8 volts/psf.

The first series of tests were based upon the transient excitation of the ceiling by jumping and by the excitation of the windows manually. There was not sufficient instrumentation available for field use to make possible the measurement of phase angles between the various elements of the system. However, some important basic information was obtained as follows; a) the vibration of one window produces a measurable pressure within the volume of the store, and b) the transient excitation of the roof by the jump method produces a measurable pressure fluctuation within the main volume of the store. The frequency of the pressure fluctuation is the same as the natural frequency of the roof. When the roof is excited by the jump method, the windows are excited at the same frequency.

It cannot be said conclusively that the motion of the windows, which is at the same frequency as the motion of the ceiling, is caused only by the pressure oscillations inside the room. There is a remote chance that structural vibrations of the roof could excite the windows. However, in view of the fact that the motion of either the window or the ceiling produces a measurable pressure oscillation within the volume of the room suggests that there is coupling between the ceiling-roof structure and the windows. It is important to note that the mullions in which the windows are installed carry none of the weight of the roof. The roof is supported by pipe columns, by curtain walls inside the store, and by the east and west masonry walls. Hence, it is unlikely that the windows are excited mechanically by transmitted vibration.

A second series of tests was run in which the roof of the store was excited by a rotating eccentric. See Figure 9 . The purpose of the test was to identify resonances and to verify the presence of acoustic coupling between the ceiling and windows.

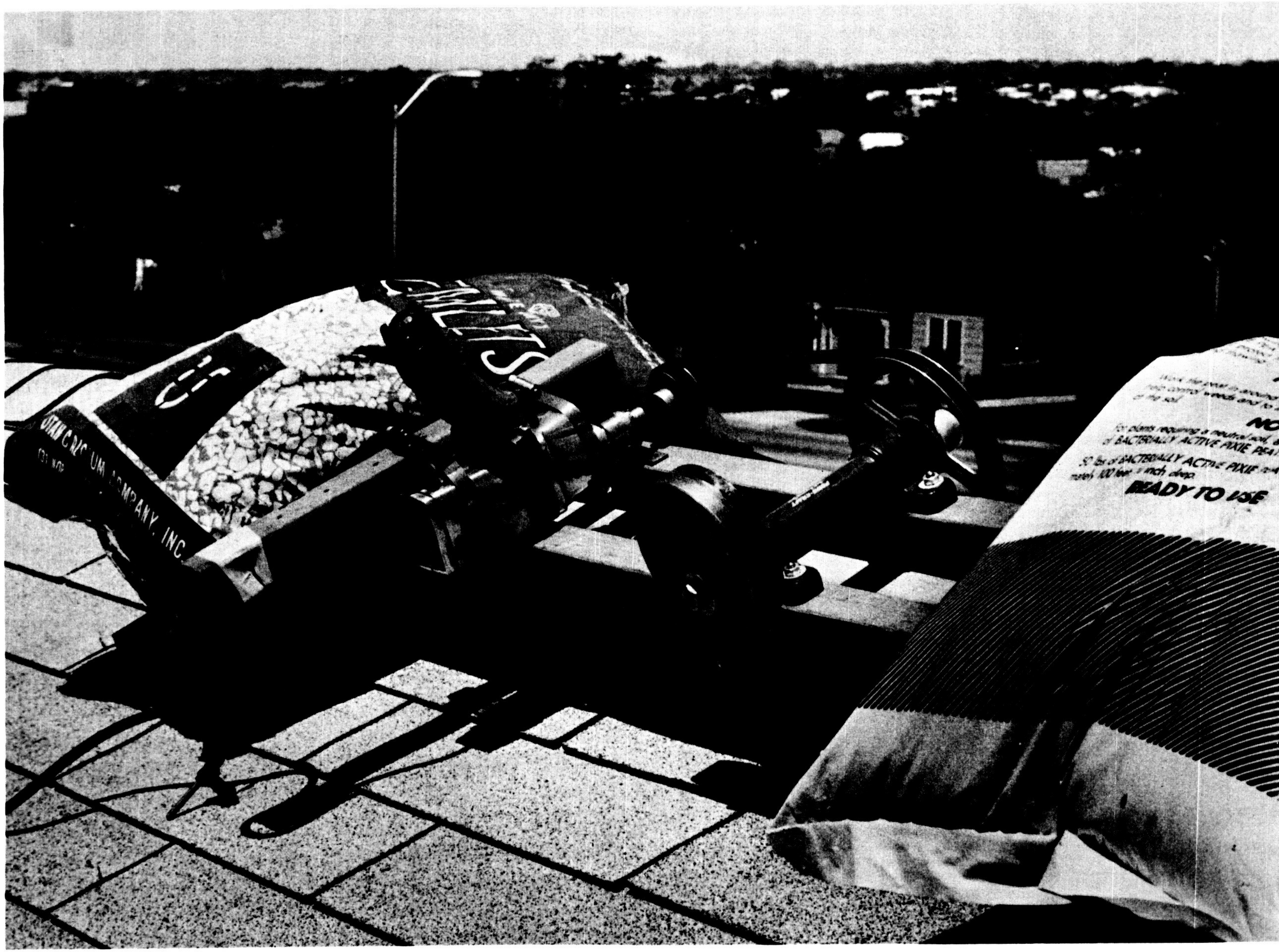


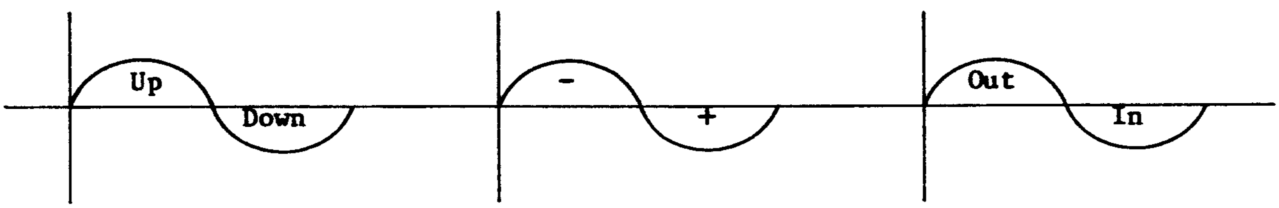
Figure 9 - Rotating eccentric device used to excite ceiling-roof structure.

The pressure was measured by a crystal microphone system having a sensitivity of 11.8 volts/psf. This is approximately 10 times the sensitivity of a standard condenser microphone system when only a cathode follower is used. The window motion was measured by differential transformer, at a point about 1.5 feet below the center, on the vertical center line. It was not possible to locate the transducer exactly at the center due to space considerations.

The motion of the ceiling was measured by an integrated velocity transducer which was calibrated in the appropriate frequency range.

Figure 10 is an oscillogram of the ceiling motion (top trace) and the pressure (bottom trace). The phase shift present can be attributed to the velocity transducer, viz. 60 degrees lag at 8 cps. The peak displacement of the ceiling is about .0025 in. and the pressure is about .017 psf, at a frequency of 9 cps.

Figure 11 is a recording of pressure (upper trace) and window motion (bottom trace). The width of the bottom trace is due to a lack of complete filtration in the discriminator of the differential transformer. The pressure is .017 psf and the motion of the window is .006 in. at a frequency of 9 cps. Figures 10 and 11 were recorded at the same time so the values can be compared. The respective polarities of the recordings are shown in sketch below. Hence, as the ceiling moves about .0025", the pressure, in phase, is .017 psf and the window moves out .006". The motion of the center of the window would be somewhat greater, probably around .007".

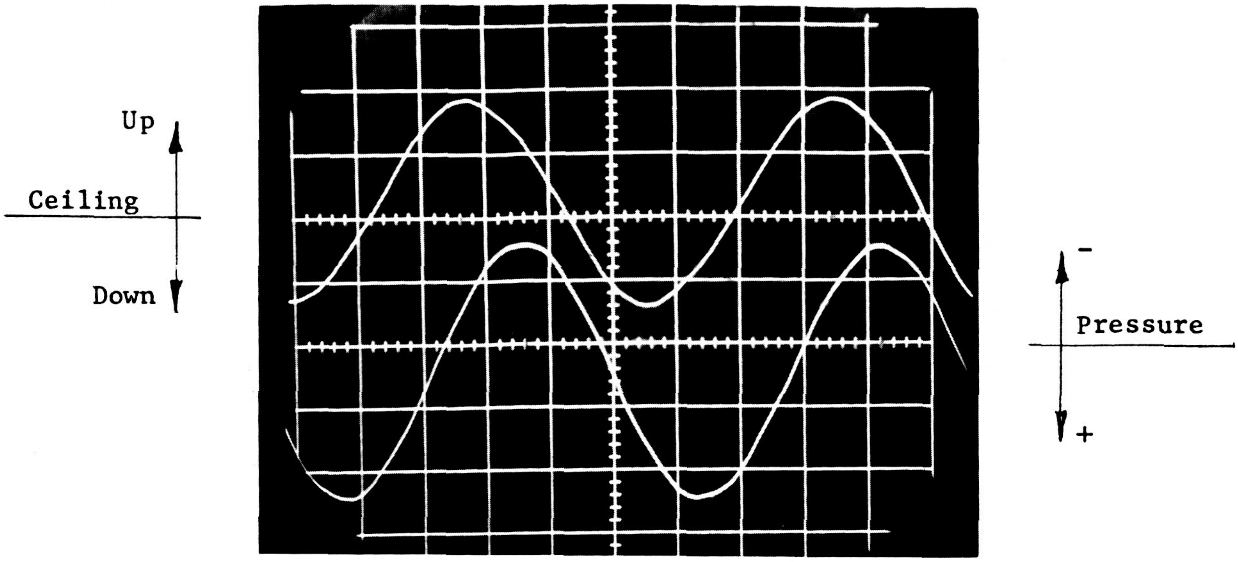


Ceiling Pressure Window

Nature of the Excitation

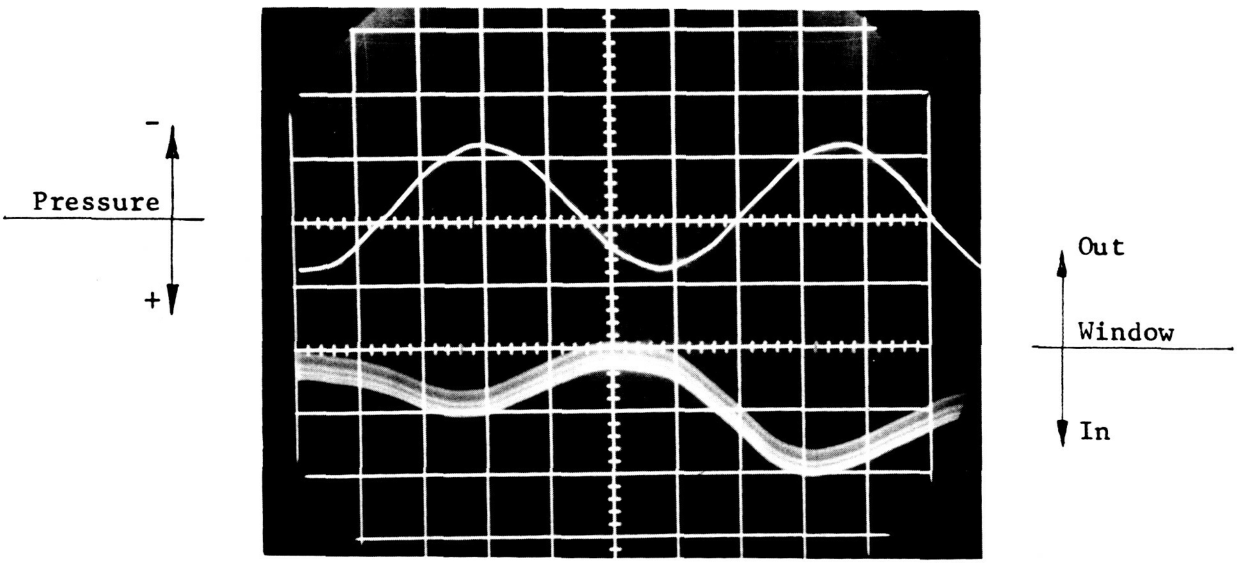
In the related analysis of the shock wave (Ref. 1) it is shown that the pressure-time history acting on Window #3 is very similar to that which would be measured at ground level. The effect of the overhanging roof is slight when compared to the possible deviations from the measured wave of Test House #1. Therefore, for the following analytical work, the wave is assumed to be a true N wave of Δt = .135 sec. and peak overpressure = 1.65 psf. Since no pressure measurements were made near the building at the time of the window failure, this assumption is the only logical one.

The assumed wave travels across the span of windows in approximately .07 second, a time which is small compared to the natural periods of any of the windows. At most, the phase difference between the identical #1 and #8 windows would be only 116 degrees, and all other phase lags would be smaller. For this reason, the load on the system is assumed to be suddenly applied on all the windows simultaneously. This should, in effect, result in greater theoretical deflections which in turn serve to identify the upper bounds of predicted stress.



Sweep = 20 milliseconds/cm

Figure 10 - Oscillogram of ceiling motion (upper trace) and interior air pressure (lower trace).



Sweep = 20 milliseconds/cm

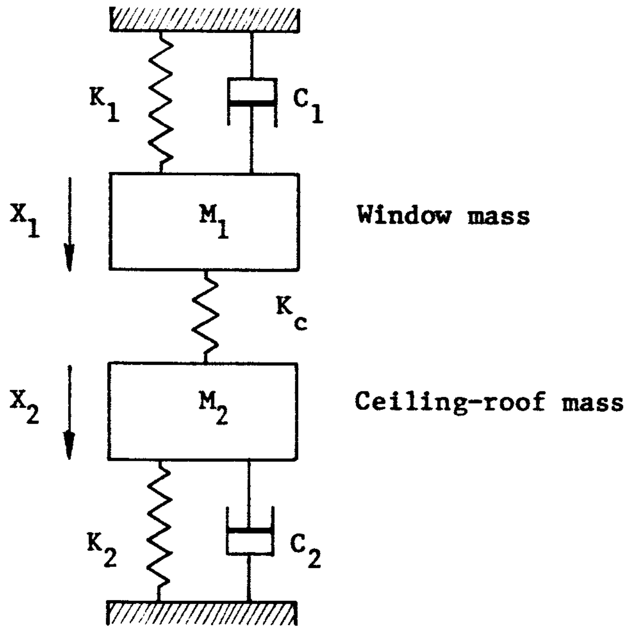
Figure 11 - Oscillogram of interior air pressure (upper trace) and window motion (lower trace).

Derivation of Differential Equations

The equivalent masses and spring constants of the windows and ceiling-roof were calculated by equating the potential and kinetic energies of the respective plates, when vibrating in the first mode, to those of a piston-spring system. This method gives the same natural frequency for both systems as well as the same maximum displacement, to a given pressure.

The approximate damping coefficients were calculated from the log decrements of the transient vibration records discussed in an earlier section. Probably some damping exists between the two masses but it is impossible to specify a value.

The basic lumped parameter model used in this study is shown in sketch below. It should be noted that K_c is a generalization of the restoring force of the trapped air in the entire building. Since the areas of the windows and ceiling are not equal, it is necessary to specify K_{c1} and K_{c2} as the respective spring constants.



K_1 is the equivalent spring of the windows

C_1 is the damping coefficient of the windows

M_1 is the effective mass of all the windows

K_c is the air elasticity

M_2 is the effective mass of the ceiling

C_2 is the damping coefficient of the ceiling

K_2 is the equivalent spring of the ceiling-roof

The differential equations of motion are:

$$M_1\ddot{X}_1 + (K_1 + K_{c1})\ X_1 + C_1\dot{X}_1 - K_{c2}X_2 = F_1(t)$$

$$M_2\ddot{X}_2 + (K_2 + K_{c2})\ X_2 + C_2\dot{X}_2 - K_{c1}X_1 = F_2(t)$$

The numerical values of the coefficients as measured and calculated are:

$$M_1 = 21.24 \text{ lbs. sec.}^2\text{/ft.}$$

$$M_2 = 310.8 \text{ lbs. sec.}^2\text{/ft.}$$

$$K_{c1} = 2{,}385 \text{ lbs./ft.}$$

$$K_{c2} = 50{,}200 \text{ lbs./ft.}$$

$$K_1 = 21{,}000 \text{ lbs./ft.}$$

$$K_2 = 786{,}000 \text{ lbs./ft.}$$

Rearranging the equations,

$$M_1\ddot{X}_1 = -\ C_1\dot{X}_1 - (K_1 + K_{c1})\ X_1 + K_{c2}X_2 + F_1(t)$$

$$M_2\ddot{X}_2 = -\ C_2\dot{X}_2 - (K_2 + K_{c2})\ X_2 + K_{c1}X_1 + F_2(t)$$

Substituting in numerical values,

$$21.24\ \ddot{X}_1 = -\ C_1\dot{X}_1 - (21{,}000 + 2{,}385)\ X_1 + 50{,}200\ X_2 + F_1(t)$$

$$310.8\ \ddot{X}_2 = -\ C_2\dot{X}_2 - (786{,}000 + 50{,}200)\ X_2 + 2{,}385\ X_1 + F_2(t)$$

$$\ddot{X}_1 = -\frac{C_1}{21.24}\dot{X}_1 - \frac{2{,}385}{21.24}X_1 + \frac{50{,}200}{21.24}X_2 + F_1(t)$$

$$\ddot{X}_2 = -\frac{C_2}{310.8}\dot{X}_2 - \frac{836{,}200}{310.8}X_2 + \frac{2{,}385}{310.8}X_1 + F_2(t)$$

$$\ddot{X}_1 = -\frac{C_1}{21.24}\dot{X}_1 - 1{,}101\ X_1 + 2{,}363\ X_2 + \frac{F_1(t)}{21.24}$$

$$\ddot{X}_2 = -\frac{C_2}{310.8}\dot{X}_2 - 2{,}690\ X_2 + 7.67\ X_1 + \frac{F_2(t)}{310.8}$$

$$\ddot{X}_1 = -\ 0.0471\ C_1\dot{X}_1 - 1{,}101\ X_1 + 2{,}363\ X_2 + 0.0471\ F_1(t)$$

$$\ddot{X}_2 = -\ 0.00322\ C_2\dot{X}_2 - 2{,}690\ X_2 + 7.67\ X_1 + 0.00322\ F_2(t)$$

Analogue Computer Solutions

The differential equations derived in the preceding section were solved by classical analogue techniques. The N waves were created by the integration and cut-off of a step function. The actual scaled recordings are shown in Figure 12. The over-shoot at the end of the wave is caused by the "Y" carriage of the X-Y recorder, and is a mechanical consideration only. The two waves have opposite signs since positive deflection is taken as downward roof motion and outward window motion.

Figure 13 is a recording of X_1X_2 for $\zeta_1 = .01$ and $\zeta_2 = .01$ and an overpressure of 1.65 psf. The maximum deflection of the window occurs on the second outswing, with a value of .277". This corresponds to an approximate stress of 470 psi at the center of the window, based upon first mode deflection.

The major significance of this record is that the peak window displacement occurs at t = .365 sec. or about .230 second after the end of the force era, indicating that energy was being fed in from the more massive ceiling-roof assembly. Since the uncoupled natural frequencies are so far apart, however, very little energy interchange can take place.

Figure 14 is a plot for $\zeta_1 = .01$, $\zeta_2 = .05$. The peak response is only slightly less (.27") but it occurs on the first outswing of the window instead of the second. It should be taken into consideration that the damping factor of the roof structure probably varies as a function of temperature and moisture content of the wood. The two values used here are probably as low as could be expected.

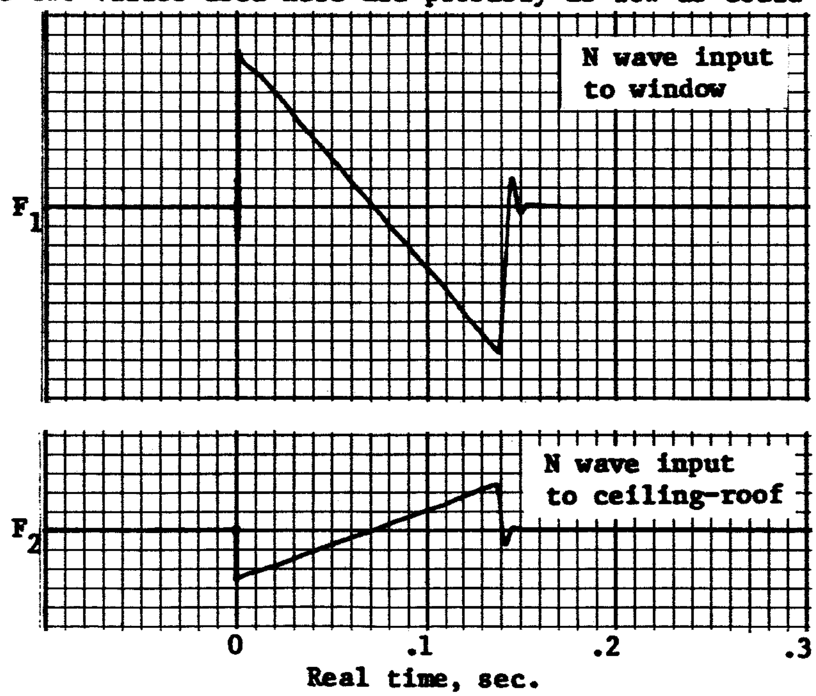


Figure 12 - N wave recordings produced for analogue computer solution.

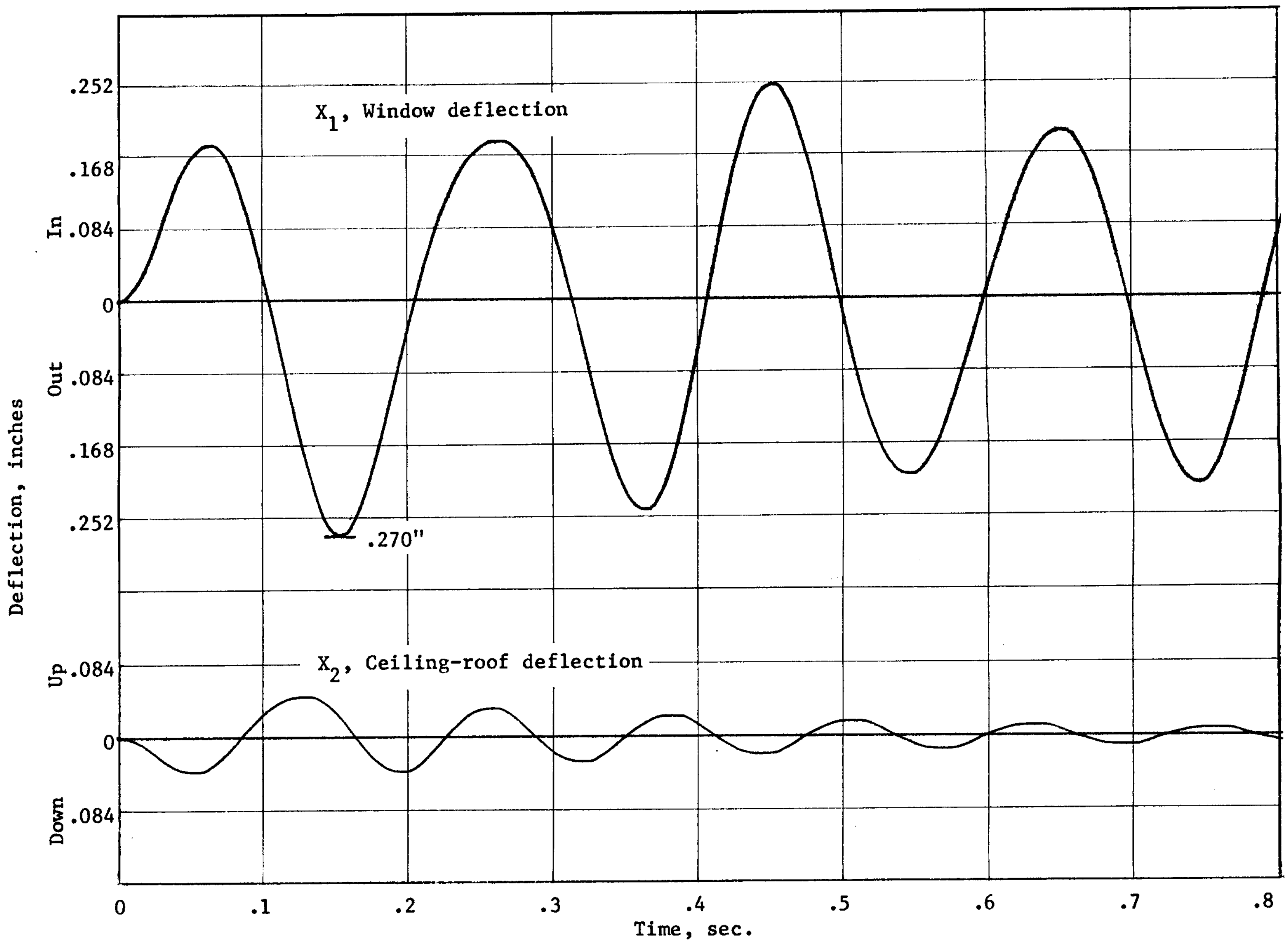


Figure 13 - Computed window and ceiling-roof response for given N wave. Damping ratio $\zeta_1 = .01$, $\zeta_2 = .05$.

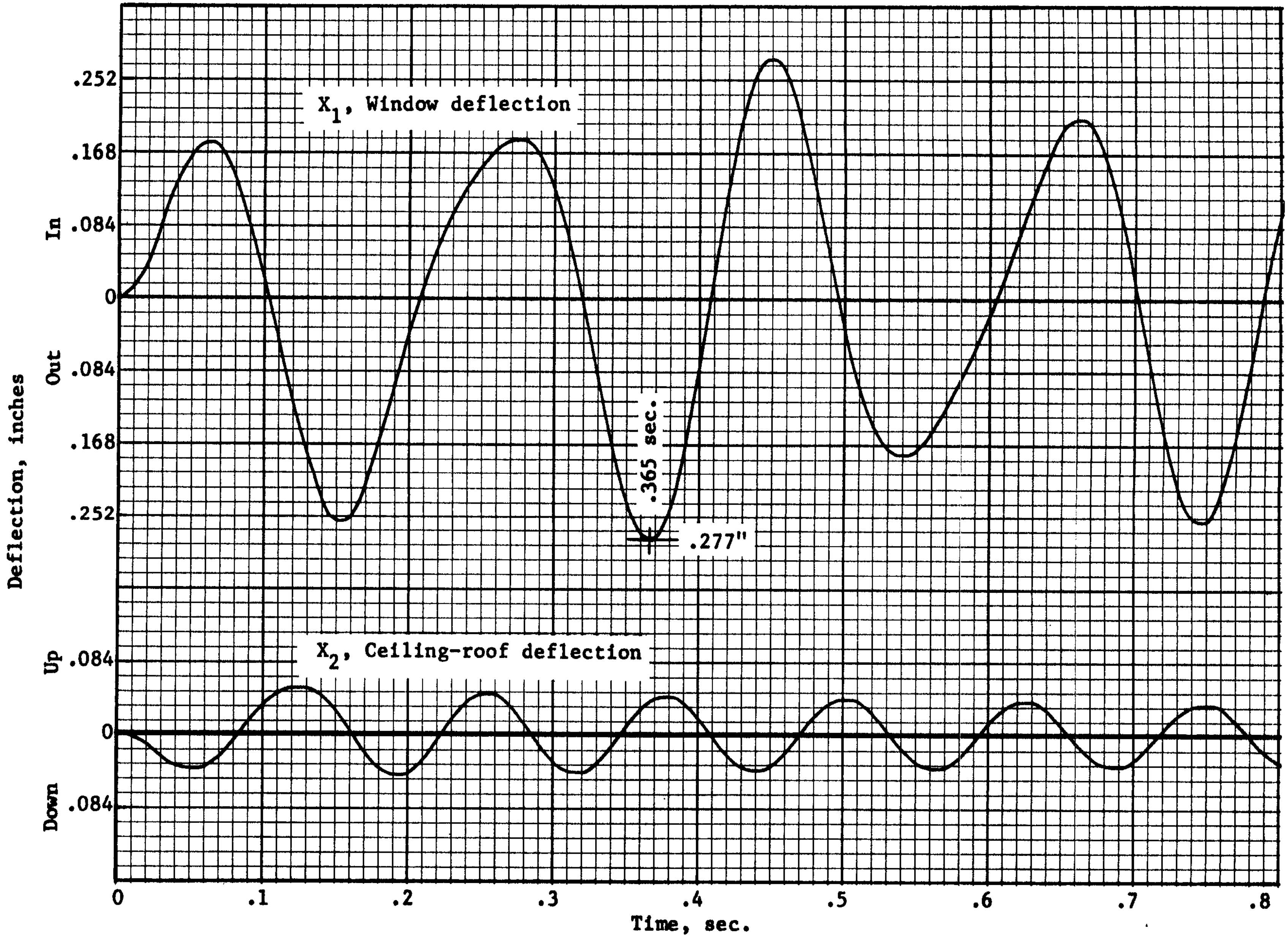


Figure 14 - Computed window and ceiling-roof response for given N wave. Damping ratios ζ_1 and ζ_2 = .01.

Digital Computer Solution for Undamped System with Two Degrees of Freedom

While the analogue computer study was concerned with only one particular model, this representation will describe several structural configurations which are of concern in sonic boom response studies. Figure 15 shows some typical structural configurations having two degrees of freedom. All of these systems can be analyzed by the same basic model described earlier, if damping is neglected.

In order to determine the maximax response of the basic two degrees of freedom system, a digital computer technique was devised. Although the calculation of maximax displacement values for a system with one degree of freedom is relatively simple, the task is amplified for two or more degrees of freedom. In these cases, the force duration of .135 second was held constant as were the two coupling frequencies. One cycle of a sine wave was used as the forcing function because it is a reasonable approximation to an N wave if only the lower vibratory modes of the system are of concern. Then the maximum response, in either the forced or residual eras, was calculated as a function of f_1, one uncoupled natural frequency. The value of f_2, the other uncoupled natural frequency, was used as a parameter. The coupling frequencies were calculated from measured data of the store building. The values of f_2 were selected to include the actual natural frequency of the ceiling-roof mass. The values of f_1 include the composite natural frequency of the windows in order to determine the effect of variations in the physical constants.

Figure 16 gives maximax values of X_1/X_{st1} and X_2/X_{st2} for the case in which the forces are 180 degrees out of phase. This corresponds to the sonic boom excitation of the building since the shock wave pushes the roof downward as the windows are pushed inward. This plot indicates that large amplitudes can be expected when the two uncoupled natural frequencies are close together and the coupling frequencies are relatively low. Also, since damping is neglected in this analysis, the upper limits of the response of the system are indicated.

Figure 17 is a plot of deflection ratios for the same system as Figure 16 except that the forces are in phase. This force configuration corresponds to a downward force of the roof and an outward force of the window. While this is opposite to the actual forcing condition, the possible condition is of mathematical interest. Since both masses are forced in the positive direction, the participation of the first mode should be strong. Also, less energy is stored in the coupling spring.

Figure 18 is a plot of deflection ratios for the same system with a force applied only to M_1. Again this force configuration is different from that of the condition under study but does represent other possible conditions. Note that deflection ratio values for this plot range from only 0 to 6 rather than the range of 0 to 12 for Figures 16 and 17 .

The sign convention used for all three plots is that a downward ceiling-roof motion is positive as is outward window motion.

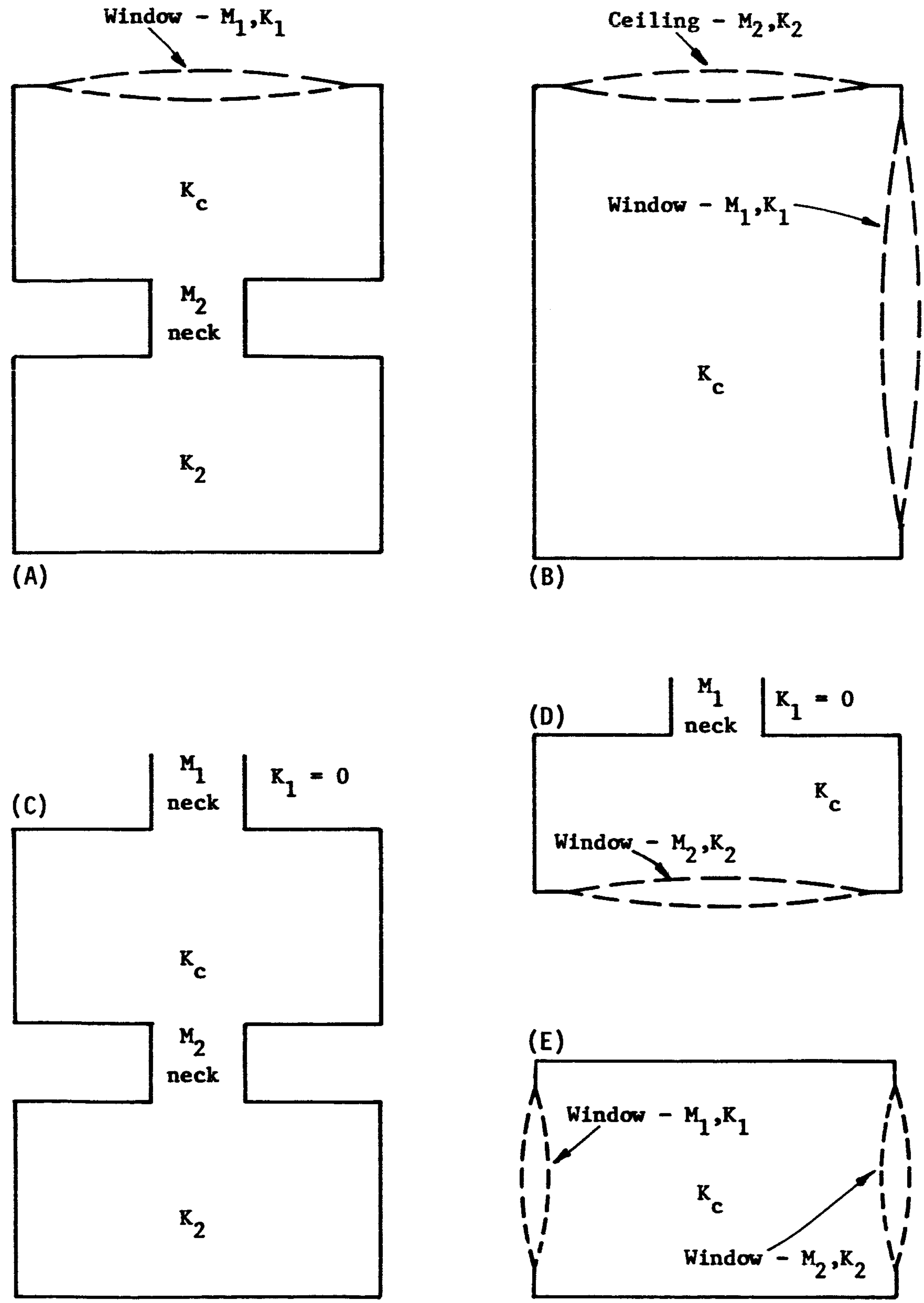
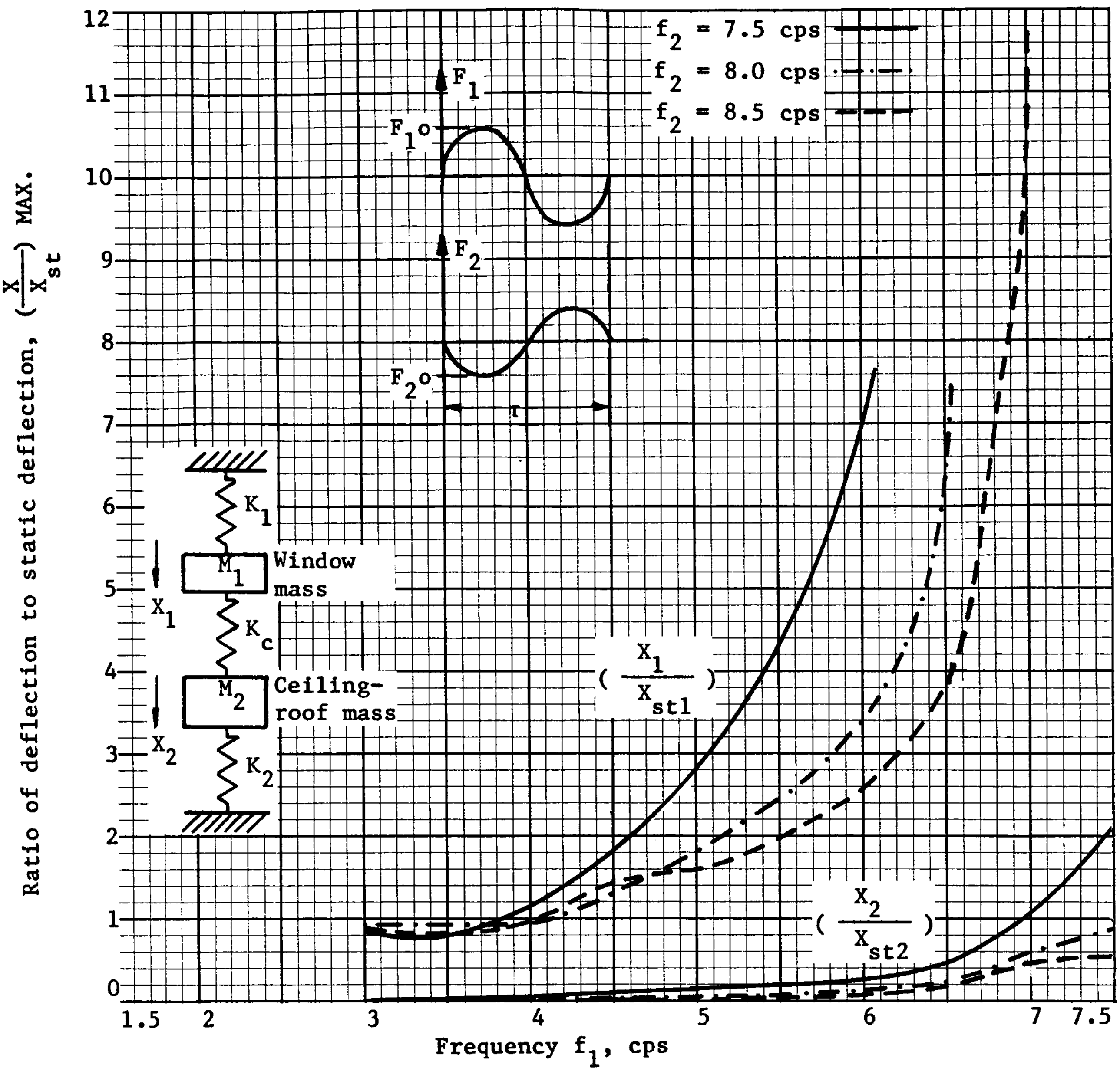


Figure 15 - Some typical structural configurations with two degrees of freedom.



τ = 0.135 sec.

f_{c1} = 1.65 cps

f_{c2} = 2.025 cps

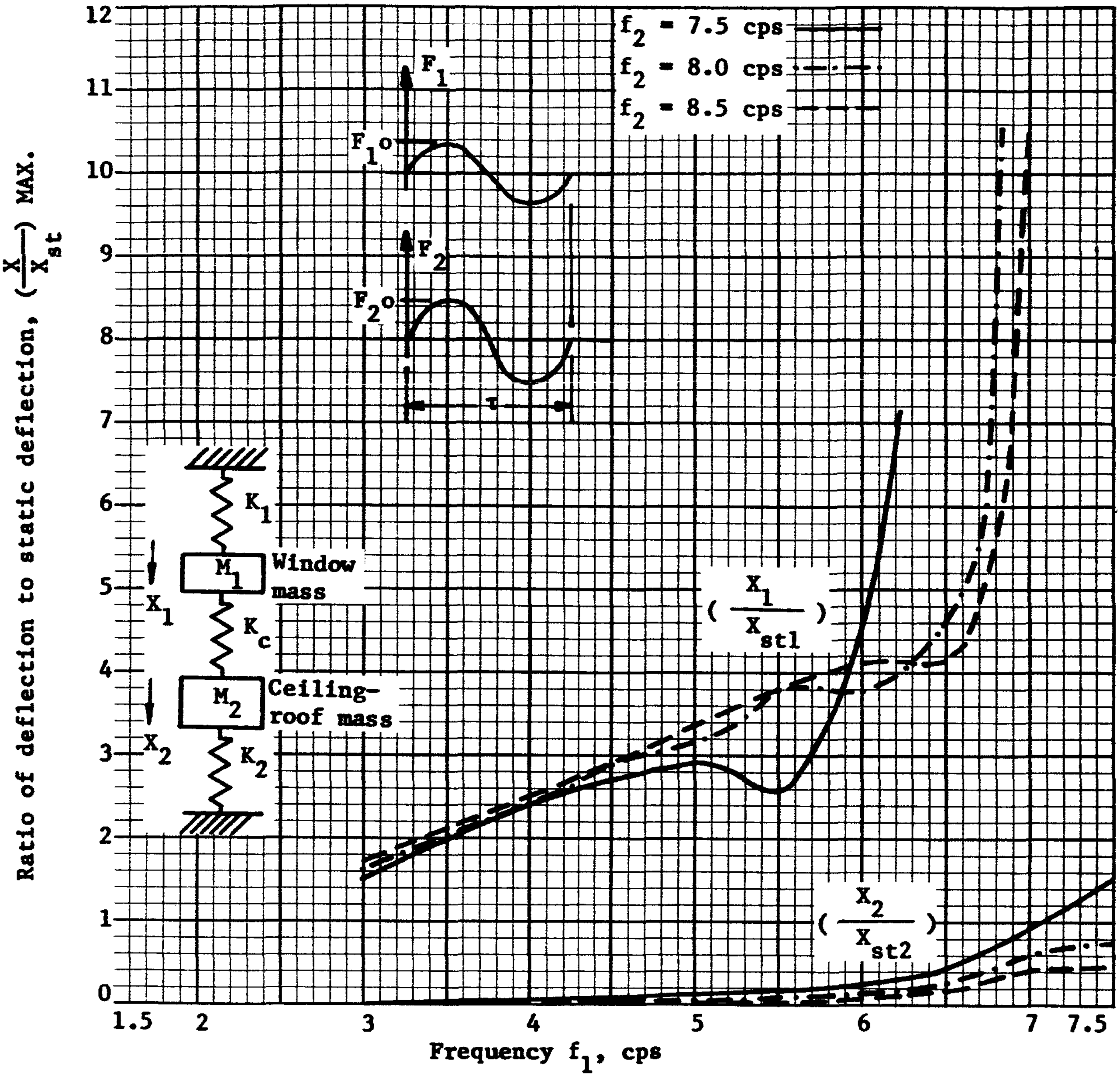
F_1o = 345.65 pounds

F_2o = 1580 pounds

$$X_{st1} = \frac{F_1o}{K_1 + K_c}$$

$$X_{st2} = \frac{F_2o}{K_2 + K_c}$$

Figure 16 - Maximum response of a generalized two degrees of freedom system excited by two forces of same time duration and 180° out of phase.



τ = 0.135 sec.

f_{c1} = 1.65 cps

f_{c2} = 2.025 cps

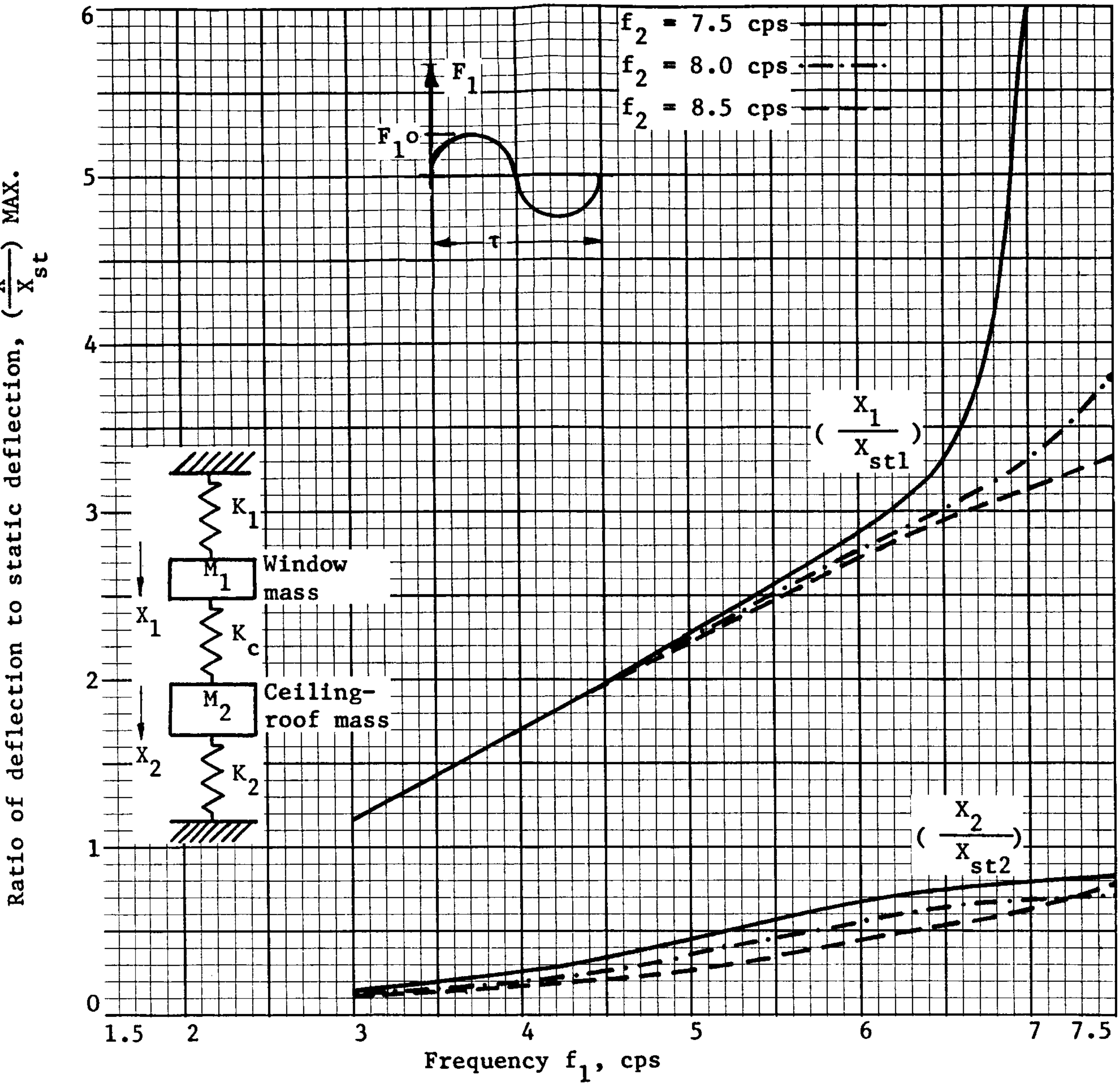
F_1o = 345.65 pounds

F_2o = 1580 pounds

$$X_{st1} = \frac{F_1o}{K_1 + K_c}$$

$$X_{st2} = \frac{F_2o}{K_2 + K_c}$$

Figure 17 - Maximum response of a generalized two degrees of freedom system excited by two forces of same time duration and in phase.



τ = 0.135 sec.

f_{c1} = 1.65 cps

f_{c2} = 2.025 cps

$$X_{st1} = \frac{F_1 o}{K_1 + K_c}$$

$$X_{st2} = \frac{F_1 o}{K_2 + K_c}$$

Figure 18 - Maximum response of a generalized two degrees of freedom system excited by only one force applied to one mass.

Digital Computer Solution for Response of a Simply Supported Plate

If acoustical coupling effects are assumed to be negligible, the response of Window #3 can be obtained by solving the classical plate equation for an N wave input at normal incidence. Several other research papers have thoroughly explored this problem.

Figure 19 is a stress vs. time history for the window for an overpressure of 1.65 psf and a damping ratio of .1. The first 25 modes were taken into consideration. The stress is greater in the y direction with a peak value of 577 psi occurring on the first outswing of the window. The plot closely resembles a pure sine wave although the presence of some higher modes can be detected. The lack of high frequency content is probably due to the low value of $\frac{\tau}{T}$.

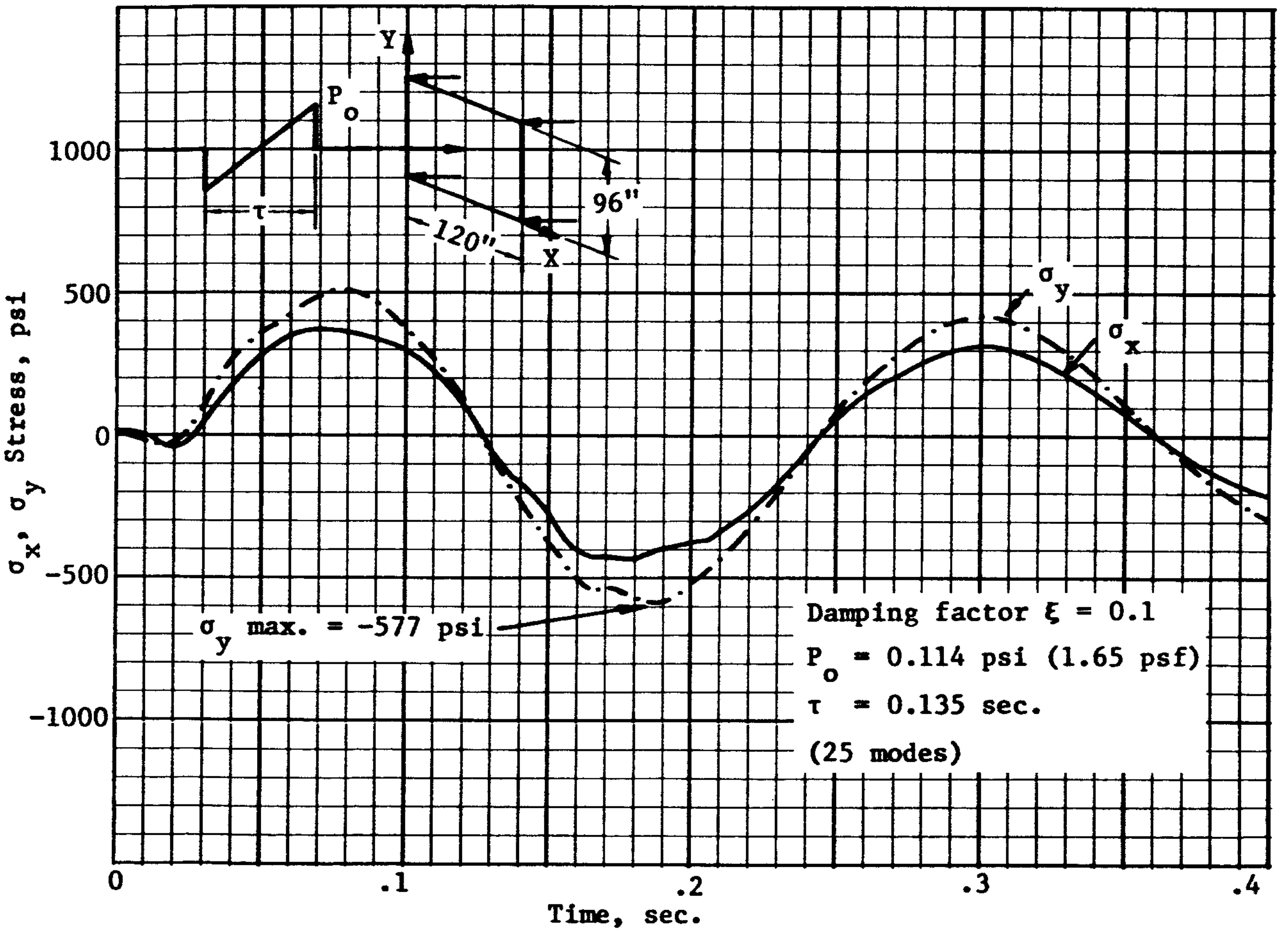


Figure 19 - Stress vs. time history at center of window for N wave at normal incidence.

DISCUSSION

In the analogue study, the resultant peak stress was found to be approximately 470 psi, which is low compared with the normally accepted value of 6,000 psi allowable stress for plate glass.

An analysis of the window as a plate in an infinite baffle produced a peak stress of 570 psi. A comparison between the two methods suggests that although acoustic coupling exists between the windows and ceiling, it is probably negligible in this case.

In the study of undamped transient response spectra, it was determined that the deflection magnification factor of the window was on the order of 1.5. This low value is a result of a lack of tuning between the ceiling and the windows. This study also shows that, under certain conditions, very lightly damped structures having uncoupled frequencies close together may exhibit very large deflection magnification factors.

Typical breaking strength of large plates of glass as glazed is 6,000 pounds per square inch for short term loadings of a fraction of a second. The strength of the same plate of glass under long term loadings of 2 hours or longer is only 3,000 pounds per square inch. These values, however, cannot be relied upon to make completely accurate predictions of damage due to pressure loadings. In one example cited by McKinley (Ref. 4), the average breaking stress for a piece of 1/4 inch polished plate glass is 8,400 pounds per square inch. For 287 specimens, a standard deviation (sigma) of 1,865 pounds per square inch is shown. The coefficient of variation is + or - .22. This indicates that for 287 specimens, three of them would break at 4,200 psi and only 37 would have the average strength of 8,400 psi.

Since there is a wide variation in strength even for specimens whose surface finish is carefully controlled, the glass industry applies a safety factor so that the few -3 and -4 sigma windows would not fail under moderate wind loadings. In the design of most windows, a safety factor of 2.5 is utilized which means statistically that 8 panes out of 1,000 could break for each initial occurrence of the design load. Using a safety factor of 2.5, Window #3 in the store building would be designed for a wind load of approximately 14 pounds per square foot which corresponds to approximately 67 miles per hour.

When the surface of a window pane is badly abraded, the maximum allowable stress is reduced up to 50%. In the case of distinct discontinuity such as holes resulting from rocks or air rifle pellets or deep surface scratches, it is impossible to say what magnitudes of stress might be encountered.

Since the condition of the window in question was not known prior to the time of failure, the ultimate stress can only be a point of conjecture. If it were assumed that the window had a bad surface condition and that it was a -3 sigma statistical sample, it could be that the relatively mild sonic boom of 1.65 psf could have broken it. However, the actual magnitude of the pressure signature which apparently broke the window is unknown. The fact that the pressure was measured at Test House #1 as 1.65 psf gives little evidence on which to base an assumption as to the pressure at the location of the store building.

It must be considered too that before the flight which apparently broke the window, the same window had been subjected to overpressure signatures resulting from B-58 flights which produced much longer signatures. The sonic boom resulting from the flight of the B-58 would in all liklihood have produced a significantly higher level of stress in this window, yet none of the B-58 flights caused failure. This gives rise to a bewildering number of possibilities to explain the failure in the glass. They are listed as follows:

1. The sonic boom had nothing to do with the failure. The glass failed from incipient thermal stresses or other causes.

2. The window had an air rifle hole or other puncture near its center causing extremely high stress concentration factors. Since glass is a brittle material not given to localized yielding, a small hole could cause very high stress levels. For this theory to be valid, it must be assumed that the hole in the window had appeared after the B-58 flights which failed to damage the window.

3. The pressure-time condition at the store location was considerably different than that measured at Test House #1 perhaps as a result of acceleration, maneuvering, or some other cause.

4. The original window was not glazed properly and glass to metal contact existed at one side or in one corner. This type of mounting can result in very high localized stresses which would precipitate failure. Also, since this is a very large window permitting large deflections of the center, if the window were not cut to fit the frame exactly, there is a possibility that a large deflection might cause one edge to come out of its mullion which, of course, would result in an increase in stress. (Note: The fact that Window #6, the left hand duplicate of Window #3, was found to be properly glazed as reported in Structural Considerations section of this report does not, of course, rule out the possibility of improper glazing of Window #3.)

Two other factors enter into the considerations of the cause of failure of the window. One is that eye witnesses said they saw the window bulge out at the time of the boom. One witness said that she was 15 feet away from the window when it shattered. See Figure 2. The engineers inspecting the damage verified that virtually all of the glass fell outside the building, which does indicate that the window was broken on an outward swing. Therefore, it seems likely that the sonic boom did have some effect in that the maximum stress would result on the outward swing of the window, as had been indicated in other basic studies. The fact that eye witnesses indicate that the window "bulged out" before its failure probably is of little importance since a very small deflection of a polished surface gives the illusion of great deflection. In fact, during the field testing phase of this study, deflections of .005" of the windows could be discerned with the naked eye from a distance of 50 feet due to the shifting reflection patterns. The most important information in the eye witness accounts is that the glass did fall on the outside of the building, and coincidentally with the sonic boom.

CONCLUSIONS

1. Acoustic coupling exists between the ceiling-roof structure and the windows of the store building but can not be considered of consequence for this particular case.

2. The stress level of Window #3 which would have been produced by the assumed 1.65 psf and .135 second duration sonic boom could not cause failure of a properly installed, undamaged window glass.

3. The natural frequency of the ceiling-roof structure changes with time and appears attributable to change in wood moisture content.

4. There are no openings of sufficient size to be classified as necks of Helmholtz resonators in this system.

5. The boundary conditions of the windows as mounted in the aluminum mullions can be considered to be simply supported.

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1330 Classen Building
Oklahoma City, Oklahoma, July 20, 1966

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2. McKinley, R. W.: Response of Glass to Sonic Booms. Paper presented at 67th Annual Meeting, American Society for Testing and Materials (Chicago, Illinois), June 1964.